Mem. S.A.It. Vol. 89, 289

Memorie della

On the PDS of GRB light curves

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Abstract. In spite of the complicated behavior in the time domain, long GRBs show a simpler behavior in the Fourier domain of frequencies, represented by power density spectra, PDS. Recently, there are some relations found between GRBs properties and PDS parameters, modeled by power-laws. Among them, the correlation between peak energy E_{peak} and PDS slope α shows a clear evidence. In this work we try to understand the origin of this correlation, making use of synthetic pulses. We find some preliminary evidences that $E_{peak} - \alpha$ relation can be seen as a new confirmation of the empiric relations $E_{peak} - L$ and $t_p - L$ for GRBs.

Key words. GRB: light-curves – PDS: Fast Fourier Transform, power-law models – Relations: Peak energy-slope

1. Introduction

The light curves of GRBs typically have many random peaks, diverse structures which appear to be the result of a complex distribution of several pulses. Burst pulses are commonly described by a fast rise exponential-decay (FRED) shape, although the decay is not strictly exponential (Norris et al. 2005):

$$c(t) = A\lambda \exp[-\frac{\tau}{t - t_s} - \frac{t - t_s}{\tau_2}] \quad (1)$$

or power-law shape (Kocevski et al. 2003):

$$c(t) = F_m(\frac{t}{t_m})^r[\frac{d}{d + r} + \frac{r}{d + r}(\frac{t}{t_m})^{r+1}]^{-\frac{r+d}{r+1}} \quad (2)$$

Analysis has shown broad log-normal distribution in time duration, not only among different bursts, but also within a single burst. In spite of extensive studies, this temporal behavior is not still understood. Light curve analysis can be a powerful tool to shed light on the still obscure physics and geometry of the prompt emission of GRBs. It can provide insights into the size, the distance of the dissipation region and the radiation processes.

In spite of the complicated behavior in the time domain, long GRBs show a simpler behavior in the Fourier domain of frequencies $c(t) \to C(f) = \int_{-\infty}^{\infty} c(t)e^{2\pi i f t}dt$.

The power density spectrum PDS is defined as $P_f = C_f C_f^*$.

In the case of observed curves with N discrete data c_m (time sequence), the discrete Fourier transform DFT is estimated by:
$C_k = \sum_{m=0}^{N-1} c_m e^{2\pi i m n/N}$, $P(f_k) = |C_k|^2$.

2. Computing PDS

One frequently used way of estimating PDS is the periodogram, given by
$P(f_k) = \frac{1}{N^2}[|C_k|^2 + |C_{N-k}|^2]$, $k = 1, 2, ..., \frac{N}{2}$.

$P(f_k)$ is considered as the average of $P(f)$ over a narrow window function, centered on f_k. This window function would naturally be $W(f) = \frac{1}{N^2}[\frac{\sin \pi f}{\sin \pi f/N}]$, the Fourier transform of the rectangular function. This window function is not zero outside the corresponding frequency interval, so the periodogram estimate is influenced from other frequencies outside the interval, technically speaking leaks from one frequency to another. The correction of the leakage is called data windowing. Instead of the rectangular function, one chooses a window function that changes more gradually from zero to its maximum and then back to zero. In our calculations, we use Bartlett function, but there are several different ones (Press et al. 1992).

Another question of the periodogram is the value of the standard deviation σ, which is 100% of the estimate, independent on the number of data N. There are several techniques for reducing the standard deviation. The technique we make use is to partition the original N data ($N = 2MK$) in $2M$ segments, each of K points. We choose a sequence of $2M$ points, one point from each segment and repeat the procedure K times, for K consecutive sampled points. Each sequence is separately Fourier transformed to produce a periodogram estimate (Press et al. 1992). Finally, the K periodogram estimates are averaged at each frequency. This final averaging reduces the standard deviation of the estimate by $\sqrt{K}$.

There are other ways used for reducing the deviation of a single PDS, one of them by using Monte-Carlo simulations of synthetic GRBs around a real one (Ukwatta et al. 2011).

3. Average and individual PDS

3.1. Average PDS

The procedure of averaging consists in summing up the PDS of individual bursts after some normalization and dividing the result by the number of bursts in the sample. The distribution of the individual P_f around $< P_f >$ follows a standard exponential law, so the amplitude of fluctuations in $< P_f >$ is given by: $\frac{<\Delta P_f>}{<P_f>} \sim N^{-1/2}$, N the number of bursts in the sample (Beloborodov et al. 2000).

There are different ways of normalization:

- the light curves are normalized to their peak (Beloborodov et al. 2000);
- the averaging is performed inside a sole group of variability, taking into account also a kind of pseudoredshift, obtained through empirical relations (Lazzati, 2002);
- the averaging is performed inside subclasses of GRBs found based on the autocorrelation function and considering the measured redshift (Borgonovo et al. 2007).

The procedure of averaging follows the conviction that different GRBs are many realizations of a unique stochastic process, giving rise to the variety of the observed profiles.

The average PDS are modeled by a power law (Beloborodov et al. 2000) or smoothly broken power-law (Guidorzi et al. 2012), extending over two frequency decades, from about 10^{-2} to 1 or 2 Hz. The power law index lies in the range $1.5 - 2$ and the break around $1 - 2$ Hz.

3.2. Individual PDS

While the average PDS over a large number of GRBs exhibits small fluctuations and is easier to characterize, it provides no clues on the variety of properties of individual GRBs. On the other hand, the wide variety of light curves exhibited by GRBs would potentially be indicative of different emission and scattering processes.

The key point of studying individual versus averaged PDS is that one can investigate the possible connection between PDS and GRBs key properties of prompt emission, such as peak energy or the isotropic-equivalent radiated energy (Dichiara et al. 2016).

As mentioned above, one way of estimating individual PDS is by simulating N light curves around the real one, by using Monte Carlo technique. The standard deviation of simulated PDS is found and the individual PDS is calculated inside this uncertainty (Ukwatta et al. 2011). This procedure follows as well the idea of a unique stochastic process inside a GRB.

Otherwise, there are authors who calculate only a single PDS over the entire observation duration (Dichiara et al. 2016), (Guidorzi et al. 2016). Each GRB time profile is considered individually as the unique sample of a unique stochastic process, which is different from other GRBs. PDS and uncertainties at each frequency are calculated assuming Leahy normalization.

To fit an individual PDS, two models are used:

- the simpler one is a mere power-law plus the white-noise constant, $S_{PL}(f) = Af^{-\alpha} + B$, A the normalization constant, α the power law index and B the white noise level;
- bent power law model, $S_{PL}(f) = A[1 + (\frac{f}{f_b})^{\alpha}]^{-1} + B$.

The power-law index lies in the range 1.5 – 4, with some exceptions to 6.

4. Some relations

There are some relations found between PDS parameters and GRBs properties. The first group of relations concerns the variability of GRBs and the second one the GRBs energy.

In the first group we could mention a correlation found between the dominant frequency and the variability measure and another correlation between the break frequency and the variability measure (Lazzati, 2002). The number of pulses also seems to anti-correlate with the slope α.

Ukwatta et al. (2011) found that the redshift corrected threshold frequency is positively correlated with the isotropic peak luminosity.

An interesting correlation is found between peak energy and the PDS power law index α of individual GRBs, $E_{peak} - \alpha$ relation (Dichiara et al. 2016).

5. A synthetic $E_{peak} - \alpha$ relation

We try to reproduce the above mentioned $E_{peak} - \alpha$ relation by generating synthetic pulses with different luminosity and by combining some empiric relations for GRBs:

- luminosity - duration relation (Hakkila et al. 2008):
  $L = 3.4 \times 10^{52} t_p^{-0.85}$,
  with t_p the pulse duration between the two Ae^{-3} intensity points, where A is the intensity at the maximum of the pulse.
- peak energy-luminosity $E_{peak} - L$ relation (Ghirlanda et al. 2005):
  $E_{peak} = 380(\frac{L}{1.6\times10^{52}})^{0.43}$
- a correlation between the rise time and pulse width measured by FWHM (Kocevski et al. 2003):
  $t_m = 0.323 t_p(1+z)^{0.6}$,
  where t_m is the pulse rise time and $t_{pobs} = t_p(1+z)^{0.6}$ is the duration of the pulse to the observer.

Assuming z=1 and normalized Band spectrum, we calculate pulse peak photon flux to the observer as (Boçi et al. 2010):

$$P = \frac{L}{4\pi D_L^2 \Phi_0 E_{peak}^{obs}} \int_{\frac{E_A}{E_{peak}^{obs}}}^{\frac{E_B}{E_{peak}^{obs}}} B(x)dx$$

Bursts pulses are described by equation 2, with $r = 1.49$ and $d = 2.39$.

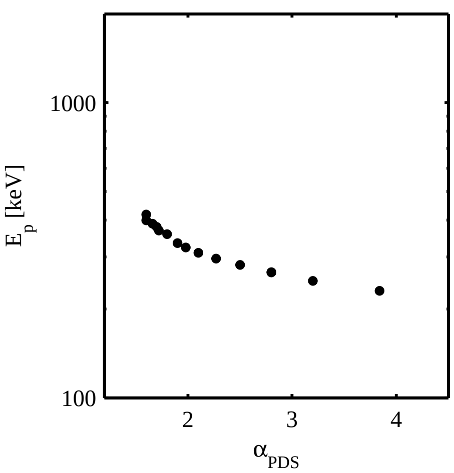

Fig. 1. $E_{peak} - \alpha$ diagram for the synthetic pulses with different luminosity. The slope α for each corresponding PDS is estimated by a power-law model, whereas E_{peak} is found based on Ghirlanda $E_{peak} - L$ empiric relation.

We calculate the PDS for each pulse and find the slope in logarithmic scale for the interval of frequencies between (0.3 – 1) Hz.

The results are plotted in Fig.1.

6. Conclusions

Although the considered sample is small, it seems that pulses with larger peak energy E_{peak} exhibit lower power-law index α, a trend similar to that found by (Dichiara et al. 2016). This result is only a preliminary one. We are performing a more systematic work by producing a larger sample of GRB's pulses. We believe that the confirmation of the $E_{peak} - \alpha$ correlation, making use of the synthetic pulses, can be seen as another evidence for empiric $E_{peak} - L$ and $t_p - L$ relations, thought to be important characteristics for using GRBs as tools in Cosmology, for scrutinizing the dark ages of the Universe, which is one of the important goals for the THESEUS mission (Amati et al. 2018). THESEUS will be very helpful to enlarge the sample of GRB ligtcurves for improving the correlations mentioned in this work.

Acknowledgements. We thank Lorenzo Amati and Cristiano Guidorzi for useful discussions on PDS, before and during Theseus workshop, Napoli, October 2017.

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